Implications of inconsistency in adjusting the cost of capital for leverage. A note.

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A common procedure for determining the cost of capital for capital investment decisions involves adjusting an unlevered beta for the risk of financial leverage. This note demonstrates that a widespread practice of levering the beta coefficients using the formula of Hamada implies a possibility of significant errors in the cost of capital estimates that may eventually yield implausible valuation results. To avoid potential distortions, the formula of Hamada should be replaced with a consistent expression relating the beta coefficients of unlevered equity, levered equity and debt.

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1. Introduction

Levering the unlevered beta coefficient to the appropriate capital structure is an integral part of the estimation of the cost of capital for a corporate division or private firm under the "pure-play" approach. It is not uncommon for textbooks and practitioners to advocate doing that by utilizing the Hamada formulation (1972) that integrates the Modigliani-Miller propositions (1963) and the

capital asset pricing model (CAPM) [1]. However, the theoretical framework developed by Hamada is not consistent with the specifications of most real-life problems a decision maker faces, and was a subject of criticism from academics in several occasions. Miles and Ezzell (1985) point out that levering beta coefficients following Hamada is invalid under a constant leverage policy, Conine (1980) proves that the Hamada formulation is not compatible with the assumption of issuing risky debt, and Arzac (2005) explains why the original results of Hamada do not hold in general. This note adds to the existing literature by explicitly demonstrating how levering the beta coefficient using the formula of Hamada when the firm's debt is risky can lead not just to an erroneous cost of capital and value estimates, but also to outcomes that are contrary to the basic corporate finance theory and common sense. The exposition and findings would be of interest to a practitioner using the cost of capital estimates for the purpose of cash flow discounting and financial performance evaluation.

2. The benchmark APV valuation

Assume a firm with an expected free cash flow *FCF*=\$90 million in perpetuity. The firm is subject to an income tax at a rate of τ=25%, the amount of debt outstanding is *D*=\$182 million, the interest rate on debt is r_D=14%, and it is equal to the market cost of debt capital k_D. The current debt level is expected to be permanent.

By substituting the supposed market data for the risk free rate R_f=5%, equity risk premium ΔR_m=13% and unlevered beta β_U=1.0 into the *CAPM*, the firm's cost of capital assuming all equity financing (i.e. the cost of unlevered equity) is estimated to be

[1] See, for example, Peterson and Fabozzi (2004), Brigham and Houston (2014), Collier et al (2007).

$$k_U = R_f + \beta_U \Delta R_m = 5\% + 1 \cdot 13\% = 18.0\% \quad (1)$$

The benchmark valuation of a firm with mixed (equity and debt) financing is provided by the Myers' (1974) adjusted present value (*APV*) rule

$$V_L = V_U + V_{TS} \quad (2)$$

where V_U is the value of the firm as if it were financed entirely by equity, V_{TS} is the value of the interest tax shields, and V_L is the value of the levered firm.

In case of perpetual *FCF*

$$V_U = FCF / k_U = 90 / 0.18 = \$500 \text{ million.} \quad (3)$$

According to Modigliani and Miller (1963), the value of the interest tax shields when the firm maintains a constant level of debt *D* and pays a tax rate τ is

$$V_{TS} = D\tau = 182 \cdot 0.25 = \$45.5 \text{ million.} \quad (4)$$

Substituting the figures obtained into the equation (2) yields $V_L = 500.0 + 45.5 = \$545.5$ million.

The value of the firm's equity V_E is a residual after the deduction of the debt claim value. Since the debt is assumed to be issued and refinanced at market terms, that is $r_D = k_D$, its book value *D* and market value V_D are equal, and, therefore

$$V_E = V_L - V_D = V_L - D = 545.5 - 182.0 = \$363.5 \text{ million.} \quad (5)$$

3. Equity residual valuation

Following a common practice, one can evaluate a firm's equity directly by discounting the cash flow to equity (*CFE*) at the cost of equity capital k_E adjusted for the risk of financial leverage.

For the perpetual cash flow

$$V_E = CFE/k_E \,. \tag{6}$$

The *CFE* is calculated by subtracting the cash flow to debt (*CFD*) from the *FCF* and adding the tax savings (*TS*) on interest:

$$CFE = FCF - CFD + TS \,. \tag{7}$$

In case of permanent debt, the firm effectively pays only the interest, therefore

$$CFE = FCF - k_D D(1-\tau) = 90 - 0.14 \cdot 182 \cdot (1-0.25) = \$70,89 \text{ million.} \tag{8}$$

To calculate k_E, we first apply the levering formula of Hamada to adjust the unlevered beta to the firm's capital structure

$$\beta_E = \beta_U \left[1 + \frac{V_D}{V_E}(1-\tau) \right] = 1 \cdot \left[1 + \frac{182.0}{363.5}(1-0.25) \right] = 1.3755 \,, \tag{9}$$

and then substitute the levered beta obtained into the *CAPM*

$$k_E = R_f + \beta_E \Delta R_m = \ 5\% + 1.3755 \cdot 13\% = 22.9\% \,. \tag{10}$$

With these estimates for *CFE* and k_E as inputs, equation (6) yields $V_E = 70.89/0.229 = \$309.6$ million. Adding the value of debt one finds the value of the levered firm

$$V_L = V_E + V_D = 309.6 + 182.0 = \$491.6 \text{ million..} \qquad (11)$$

4. FCF-WACC valuation

Next, let us consider another common valuation technique, that is discounting the firm's *FCF* at *WACC*. Under the assumptions of our test example

$$V_L = FCF/WACC\,. \qquad (12)$$

The discount rate *WACC* is calculated via the standard formula

$$WACC = \frac{V_E}{V_L}k_E + \frac{V_D}{V_L}k_D(1-\tau)\,. \qquad (13)$$

Substituting the benchmark (market) values of V_L and V_E from the section 2, and k_E available from the section 3, obtain

$$WACC = \frac{363.5}{545.5}22.9\% + \frac{182.0}{545.5}14.0\%(1-0.25) = 18.8\%\,,$$

$$V_L = 90.0/0.188 = \$478.7 \text{ million.}$$

The value of equity under the *WACC* approach is as follows

$$V_E = V_L - D = 478.7 - 182.0 = \$296.7 \text{ million.}$$

5. Perplexing controversy of results

The observations from the test case analysis are straightforward. Three conventional cash flow discounting approaches applied to evaluate one and the same firm under the identical assumptions produce three pairs of values for the firm and its equity that do not match. This outcome is counterintuitive, since all three approaches are fundamentally equivalent (Fernández, 2007) and in the test case proceed from the same input data.

The value of equity \$309.6 million obtained by discounting the *CFE* at k_E is 14.8% lower than the consistent benchmark equity value of \$363.5 million. The error in the equity value estimate of \$296.7 million under the WACC approach increases to 18.4% with respect to the same benchmark. The errors are significant, and it is important to note, that these and the other distortions specified further in this section by no means should or could be attributed to the uncertainty in the input data. They follow from the inconsistencies embedded in the way the cost of equity capital is adjusted for leverage.

The figures for the levered firm value arising from the equity residual and *WACC* methods, \$491.6 and \$487.7 million respectively, do not match either. In addition, both are lower than the value of the unlevered firm $V_U = \$500$ million. The latter is confusing, since it implies a negative value of the interest tax shields.

Overall, the test case presented makes it perfectly clear that a widely adopted cash flow valuation procedures incorporating Hamada's relevering formula may produce outcomes contrary to the fundamental principles of corporate finance laid by Modigliani and Miller, as well as to common sense.

6. Restoring coherence in valuation

Recall that the original results of Hamada (1972) were based upon the Modigliani-Miller's (1963) tax shield value $V_{TS} = \tau D$ for a constant debt and the companion assumption that the debt is risk free (i.e. $\beta_D = 0$). When the debt is risky, its systematic risk makes β_D positive, and while the Modigliani-Miller's result still holds, the Hamada's formula becomes void. As an inescapable implication of a risky debt, the way the cost of equity is calculated to discount *CFE* and estimate *WACC* has to be changed.

According to Conine (1980), the correct formula to be applied for levering beta in case of a risky permanent debt is

$$\beta_E = \beta_U + \frac{V_D}{V_E}(\beta_U - \beta_D)(1-\tau). \tag{14}$$

The cost of debt could be estimated by applying the *CAPM*:

$$k_D = R_f + \beta_D \Delta R_m. \tag{15}$$

Going back to numbers one obtains

$$\beta_D = (0.14 - 0.05)/0.13 = 0.6923,$$

$$\beta_E = 1.0 + \frac{182.0}{363.5} \cdot (1.0 - 0.6923) \cdot (1 - 0.25) = 1.1155,$$

$$k_E = 5.0\% + 1.1115 \cdot 13.0\% = 19.5\%,$$

$$WACC = \frac{363.5}{545.5} \cdot 19.5\% + \frac{182.0}{545.5} \cdot 14.0\% \cdot (1 - 0.25) = 16.5\%,$$

$$V_E = 70.89/0.195 = \$363.5 \text{ million},$$

$$V_L = 90.0/0.165 = \$545.5 \text{ million}.$$

Thus, the equity residual and the *WACC* approaches yield identical figures, and both match, as they should, the benchmark *APV* valuation. The use of an appropriate relevering formula eliminates inconsistences encountered earlier.

Note, that the beta estimates, and hence the cost of equity and *WACC* figures produced from utilizing the Hamada formulation are significantly higher than the consistent estimates. This observation conforms to the empirical findings of Fuller and Kerr (1981).

7. Concluding remarks

A classic approach to adjust the cost of capital for leverage pioneered by Modigliani and Miller and elaborated on by Hamada invokes two assumptions: (a) the firm is able to borrow and lend at a risk-free rate; and (b) it maintains permanent the amount of debt outstanding. Although these assumptions are apparently unrealistic, the approach persists in the capital budgeting and valuation schemes. This note demonstrates how a failure to comply with just the first assumption, which is a common situation in the real life applications, may cause significant bias in the cost of capital estimates and implausible valuation outcomes contrary to the foundations of corporate finance. Therefore, a strong argument is made to abandon the practice of using the formula of Hamada in favor of a not so popular, but consistent levering expressions developed in the literature (Taggart 1991, Inselbag and Kaufold 1997, Tham and Vélez-Pareja 2004) for different financing policies under realistic assumptions.

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